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Starting from an (unknown) quantum gravitational model, one can invoke a sequence of approximations to progressively arrive at quantum field theory (QFT) in curved spacetime, QFT in flat spacetime, nonrelativistic quantum mechanics and newtonian mechanics. The more exact theory can put restrictions on the range of possibilities allowed for the approximate theory which are not derivable from the latter – an example being the symmetry restrictions on the wave function for a pair of electrons. We argue that the choice of vacuum state at low energies could be such a ‘relic’ arising from combining the principles of quantum theory and general relativity, and demonstrate this result in a simple toy model. Our analysis suggests that the wave function of the universe, when it describes the large volume limit of the universe, dynamically selects a vacuum state for matter fields — which in turn defines the concept of particle in the low energy limit. The result also has the potential for providing a concrete quantum mechanical version of Mach’s principle.

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It is a well known fact that our knowledge about the fundamental laws of physics becomes increasingly uncertain, as we proceed to higher energies. A fortunate feature about Nature seems to be that the high energy behaviour of physical systems do not influence the low energy predictions based on effective hamiltonians, allowing a steady progression of our understanding through a sequence of effective theories. This behaviour, which can be formalised in terms of renormalisation group analysis, forms the cornerstone of quantum field theoretical description of Nature at low energies.

There are, however, some key features of high energy phenomena which leave traces at low energies as regards the restrictions on allowed quantum states. For example, the Pauli exclusion principle plays a vital role in low energy atomic physics but cannot be derived or even related to any feature of low energy hamiltonian – its origin lies in relativistic field theory. Within the framework of low energy theory, there is no symmetry restrictions on the quantum state for, say, a system of two electrons. But if the electrons are treated as excited states of an underlying fermionic field, it is possible to prove that only a subset of quantum states — which are antisymmetric — are allowed. The importance of such ‘relic principles’ lies in providing a glimpse of the unknown territory.

This prompts us to ask: Is there any such effect or selection principle which arises from the high energy theory that combines general relativity and quantum theory but leaves a trace at low energies? We argue in this Letter that the choice of the vacuum state of low energy could itself be a relic of an intrinsically quantum gravitational principle.

We begin by highlighting certain issues which arise in the definition of vacuum state (and one-particle state) in the low energy theory when viewed along conventional lines. In nonrelativistic, newtonian, mechanics the concept of a free particle moving with uniform velocity in an

inertial frame, also presupposes the existence of a unique *time* coordinate. The equation  $d^2\mathbf{x}/dt^2 = 0$  is invariant only under the linear transformation  $t \rightarrow \alpha t + \beta$ . This feature continues in nonrelativistic point quantum mechanics. At the next level, combining the principles of special relativity with quantum theory extends the accepted range of time coordinate to that of any inertial observer given by standard Lorentz transformations. This class allows one to define a set of positive frequency modes, creation and annihilation operators and the inertial vacuum state  $|0\rangle_I$ . Conventional QFT works at this level and the choice of vacuum state is now unique because the class of inertial frames form a privileged set in special relativity.

This description, however, is unacceptable both conceptually and technically at a fundamental level. Conceptually, there is no reason to attribute a special status to any one class of observers — a point forcefully emphasised by Einstein while motivating general relativity [1]. Technically, any quantum field generates a gravitational field around it thereby curving the spacetime; a description of QFT in flat spacetime can only be an approximation and no fundamental feature of the theory (like the choice of vacuum state or definition of particles) should depend on the approximate model. Doing QFT in curved spacetime is *not* a matter of choice but *is mandatory, because all fields curve the spacetime*. This changes the symmetry group to that of arbitrary coordinate transformations and extends the allowed choices of time coordinate. The uniqueness of vacuum state (or the definition of particle) is lost when we work in the limit of QFT in a general curved spacetime [2].

Even this situation, however, is unacceptable because we cannot really address the issue of ground state when gravity is treated as *c*-number field and the matter field is quantised. One should really consider the full Hilbert space of the quantum theory of matter field coupled to

gravity, (we are essentially interested in the self gravity of matter; but the formalism remains the same) and try to define a ground state for this theory. The structure of such a theory is at present unknown but the indications are that the ground state wave functional will have an extremely complex structure – since it incorporates the arbitrarily high energy virtual modes of not only the matter fields but also gravity (which is nonlinearly coupled to itself). If the fundamental description is in terms of strings or spin networks, it is not even clear how to address the question of ground state in the language of conventional QFT [3]. But the above arguments clearly suggest that the choice of ground state for any quantum field theory is linked with very high energy phenomena.

It is possible make some progress if we observe that the issue of ground state has implications for cosmology. Note that an arbitrary choice of the time coordinate, even in newtonian mechanics, will make a free particle move in an accelerated trajectory. The result persists even in QFT done in a noninertial frame. It is easy to show that when the nonrelativistic wave function corresponding to the particles defined in such a coordinate system is taken, it will have pseudopotential term giving rise to an accelerated motion [4]. For example, let us consider a particle, of mass  $m$ , described by a quantum field  $\hat{\phi}$  in flat spacetime. Let  $|0\rangle_I$  and  $|1_{\mathbf{k}}\rangle_I$  denote the vacuum and one-particle states for the field in the inertial frame. The nonrelativistic limit ( $c \rightarrow \infty$ ) of the QFT can be obtained by identifying the quantity  ${}_I\langle 0|\phi|1_{\mathbf{k}}\rangle_I$  with the Schrödinger wave function  $\psi$ . It can then be shown that  $\psi$  obeys the free particle Schrödinger equation. However, if we work in a noninertial frame, say, the Rindler frame, given by the metric

$$ds^2 = \left(1 + \frac{gx}{c^2}\right)^2 c^2 d\tau^2 - dx^2 - dy^2 - dz^2, \quad (1)$$

then the particle states  $|0\rangle_R$  and  $|1_{\mathbf{k}}\rangle_R$ , defined using the Rindler time coordinate  $\tau$ , will not be the same as those defined using the inertial time  $t$ . In the nonrelativistic limit, the function  ${}_R\langle 0|\phi|1_{\mathbf{k}}\rangle_R$  has to be identified with the wave function  $\psi$ , which will satisfy the Schrödinger equation for a uniformly accelerated particle [4]

$$i\hbar \frac{\partial \psi}{\partial \tau} = -\frac{\hbar^2}{2m} \nabla^2 \psi + mgx\psi. \quad (2)$$

In the newtonian limit ( $\hbar \rightarrow 0$ ), this equation will describe a particle moving under a pseudopotential  $gx$ . Hence, if we choose our time coordinate as the Rindler time  $\tau$ , the particles (defined using  $\tau$ ), in the nonrelativistic limit, will experience a pseudo force. To realise that there is a pseudo force, one has to introduce some kind of “fixed frame of stars” as is usually done in discussions of Mach’s principle which — in a more sophisticated form — will be connected to the boundary conditions on the wave function describing the state of the universe [5].

To see this explicitly, we shall assume that the conventional approach to quantum cosmology (say, based on Wheeler-DeWitt equation) does interface between the fully quantum gravitational description of spacetime (say, in terms of spin networks or strings) and conventional QFT. Then the question of ground state translates to finding an acceptable solution to Wheeler-DeWitt equation,  $\Psi(g, \phi)$ , depending on both gravitational and matter variables denoted symbolically as  $(g, \phi)$ . Conventionally, Wheeler-DeWitt equation is solved in a FRW minisuperspace, in which a choice of time coordinate is already made — which defeats our purpose. To test our conjecture that quantum cosmological solution can effect a choice of vacuum state we need to find a sufficiently general but yet tractable approximation to Wheeler-DeWitt equation. Fortunately, this can be done along the following lines:

The least amount of dynamical structure needed to illustrate our idea is provided by the Bianchi Type I minisuperspace, with the metric

$$\begin{aligned} ds^2 = & e^{-6\Omega} dt^2 \\ & - e^{-2\Omega} [e^{2(\beta_+ + \sqrt{3}\beta_-)} dx^2 \\ & + e^{2(\beta_+ - \sqrt{3}\beta_-)} dy^2 \\ & + e^{-4\beta_+} dz^2]. \end{aligned} \quad (3)$$

The classical dynamics of the Bianchi Type I empty universe is described by the Kasner solutions [6]

$$\Omega \propto t, \quad \beta_+ = C_+ \Omega, \quad \beta_- = C_- \Omega \quad (4)$$

with the constraint

$$C_+^2 + C_-^2 = 1. \quad (5)$$

There is an equivalent description of the empty Bianchi Type I universe for which the metric has the form [6]

$$ds^2 = dT^2 - T^{2p_1} dx^2 - T^{2p_2} dy^2 - T^{2p_3} dz^2, \quad (6)$$

with the constraints

$$p_1 + p_2 + p_3 = p_1^2 + p_2^2 + p_3^2 = 1. \quad (7)$$

The time coordinates are related by

$$dT = e^t dt, \quad (8)$$

where we have chosen the scale of  $t$  in such a way that  $\Omega = -t/3$ , while the quantities  $p_1, p_2, p_3$  are related to  $C_+, C_-$  through the relations

$$\begin{aligned} p_1 &= \frac{1}{3}(1 - C_+ - \sqrt{3}C_-); \\ p_2 &= \frac{1}{3}(1 - C_+ + \sqrt{3}C_-); \\ p_3 &= \frac{1}{3}(1 + 2C_+). \end{aligned} \quad (9)$$

There is also a special solution where  $p_1 = p_2 = p_3 = 0$ , which describes the Minkowski metric.

The solutions given by equations (4) and (5), in general, represent curved spacetimes, even though they are source free. However, there exist a subset of flat spacetime solutions among them which we shall concentrate on for our illustration. The key point to note is that there are *two distinct classes of flat spacetimes*. The first class is given by the three solutions

$$\text{Class I: } \begin{cases} (i) C_+ = -\frac{1}{2}, C_- = -\frac{\sqrt{3}}{2}; \\ (ii) C_+ = -\frac{1}{2}, C_- = \frac{\sqrt{3}}{2}; \\ (iii) C_+ = 1, C_- = 0. \end{cases} \quad (10)$$

which are the flat Milne universes [7] and differ only in the choice of spatial direction along which expansion takes place. The solutions in terms of  $p_1, p_2, p_3$ , are given by

$$\begin{aligned} (i) & p_1 = 1, p_2 = 0, p_3 = 0; \\ (ii) & p_1 = 0, p_2 = 1, p_3 = 0; \\ (iii) & p_1 = 0, p_2 = 0, p_3 = 1. \end{aligned} \quad (11)$$

The second class corresponds to the choice

$$\text{Class II: } \Omega = \beta_+ = \beta_- = 0, \quad (12)$$

which describes the flat Minkowski universe. The classical dynamics of this system [given by equations (4) and (5)] can be described geometrically in the minisuperspace for this metric, which is the  $(\Omega, \beta_+, \beta_-)$ -space. The Kasner solutions are straight lines which lie on a “light cone” like structure. The three Milne flat solutions are three straight lines on this light cone separated by  $120^\circ$ . *The Minkowski flat universe, on the other hand, lies at the origin.* Since we have three variables  $(\Omega, \beta_+, \beta_-)$  here, we can choose  $\Omega$  as our time coordinate. The other two variables can then act as our dynamical variables.

The key point to note is the following: The two distinct class of solutions described above in equations (10) and (12) both correspond to flat spacetimes but differ in the choice of time coordinate. In quantum theory, they would correspond to different choices of vacuum states [8]. By constructing the necessary quantum description we can investigate how the wave function of the universe behaves vis-a-vis the choice of ground state.

To describe the quantum dynamics, we start with the Wheeler-Dewitt equation, which takes a particularly simple form for this metric [9]:

$$\frac{\partial^2 \psi}{\partial \Omega^2} - \frac{\partial^2 \psi}{\partial \beta_+^2} - \frac{\partial^2 \psi}{\partial \beta_-^2} = 0. \quad (13)$$

As usual, the Wheeler-DeWitt equation is “timeless”, but we can take  $\Omega$  as our a fiducial evolutionary parameter. The relevant solution for this equation, which gives a positive definite probability density, and also describes an expanding universe, can be written in the form

$$\psi(\Omega, \mathbf{r}) = \int K_+(\mathbf{p}) e^{i(\mathbf{p} \cdot \mathbf{r} - |\mathbf{p}| \Omega)} d^2 p, \quad (14)$$

where we have introduced the two-dimensional vectors

$$\mathbf{r} = (\beta_+, \beta_-), \quad \mathbf{p} = (p_+, p_-). \quad (15)$$

Since equation (13) is second order in time, the probability density  $|\psi(\Omega, \mathbf{r})|^2$  is, in general, not positive definite. This problem is well known for Wheeler-Dewitt equation, and is discussed by several authors (see [9]). In this work, we need not worry about this issue because the probability for the solution (14) is always positive. Let us assume that  $|\psi(\Omega, \mathbf{r})|^2$  is peaked around the Minkowski vacuum state initially. This means we can choose

$$\psi(0, \mathbf{r}) = f(\mathbf{r}) e^{i\mathbf{q} \cdot \mathbf{r}} \quad (16)$$

where  $f(\mathbf{r})$  is peaked around  $\mathbf{r} = 0$ . As  $\Omega$  increases, the wave function will spread, and the peak will move. At some sufficiently large  $\Omega$ , the probability distribution,  $|\psi(\Omega, \mathbf{r})|^2$  will be peaked around one of the classical trajectories, propagated along the characteristics of equation (13). The direction of  $\mathbf{q}$  will decide the specific classical solution around which the probability will be peaked. At some given  $\Omega$ , the probability will be peaked around the point

$$\mathbf{r} = \frac{\mathbf{q}}{|\mathbf{q}|} \Omega. \quad (17)$$

In the case where  $\mathbf{q} = 0$ , the probability, instead of peaking around any specific Kasner solution, will be peaked around the whole class of Kasner solutions, i.e., for a given  $\Omega$ , it will be peaked around the circle  $r^2 \equiv \beta_+^2 + \beta_-^2 = \Omega^2$ . Hence, it can be concluded that the quantum solutions at large  $\Omega$  will always deviate from the Minkowski universe.

In order to describe the ground state let us concentrate on the flat spacetime solutions. Hence we choose our initial conditions (i.e., the direction of  $\mathbf{q}$ ) in such a way that when  $\Omega > 0$ , the probability is peaked around any one of the Milne flat solutions. We thus see that even if we localise the wave function around the Minkowski universe initially, it will be localised around one of the Milne solutions at later stages. The quantum wave function described here tend to pick up the Milne universe at late times – hence the Milne time coordinate is preferred over the Minkowski one. We can now introduce matter fields with spatial degrees of freedom, take the limit of semiclassical gravity [10] to obtain QFT in curved spacetime and proceed to low energy limit. The Wheeler-DeWitt equation for the full system is nothing but the hamiltonian constraint equation

$$(H + H_{\text{matter}})\Psi(g, \phi) = 0. \quad (18)$$

We write  $\Psi(g, \phi)$  as

$$\Psi(g, \phi) = R(g, \phi) \exp\left[\frac{i}{\hbar} S(g, \phi)\right], \quad (19)$$

substitute it in the hamiltonian constraint equation and take the limit  $\hbar \rightarrow 0$ . Now, if we identify the derivative of the action  $S$  with respect to the gravitational variables (written symbolically as  $\delta S/\delta g$ ) with the canonical momentum, we will get the usual dynamical equations for free gravity, and, in addition,  $R$  will satisfy the Schrödinger equation

$$i\hbar \frac{\delta S}{\delta g} \frac{\delta R}{\delta g} = H_{\text{matter}} R. \quad (20)$$

This equation represents the QFT in curved spacetime. We can identify the time coordinate  $\tau$  through the derivative of the action  $S$

$$\frac{d}{d\tau} = \frac{\delta S}{\delta g} \frac{\delta}{\delta g}. \quad (21)$$

It is obvious that, in our case, the vacuum state and particles will be defined with respect to Milne time coordinate.

This simple – but adequate – example illustrate the idea we suggested earlier: It is possible that the wave function of the universe, when it describes the large volume limit of the universe (in our case large  $\Omega$  limit), dynamically selects a vacuum state for matter fields — which in turn defines the concept of particle in the low energy limit.

This particular example leads to a vacuum state corresponding to anisotropic expansion, which is — of course — not a proper description of low energy physics. But it should clear that this particular feature is an artifact of the minisuperspace we have chosen. We stress the fact that the general arguments given in this Letter are of far greater validity than the simplified example given above (which could be objected to at several levels). To emphasise the broader picture we conclude with a summary of the key ingredients of our analysis: (1) All quantum fields curve the spacetime around them due to self gravity. Hence, QFT in a flat spacetime is an approximation. (2) The choice of Minkowski (inertial) time coordinate ceases to exist in a curved spacetime. Thus the true ground state of any field is nontrivial when we realise that it generates a gravitational field around it. We need to ask whether the full quantum theory of gravity plus matter fields puts restrictions on the choice of possible ground states. The analogy will be with the allowed quantum states for a pair of electrons in atomic physics. Nothing in low energy theory prevents the existence of a state with arbitrary symmetry; but when we treat electrons as low energy excitations of an underlying fermionic field theory, we find that only antisymmetric states are allowed. (3) Since the ground state wave function of universe also figures prominently in the description of quantum cosmological models of the universe, there arises the interesting possibility of connecting the definition of free particle at low energies with the geometry of the universe,

thereby providing a quantum version of Mach's principle. (4) A toy model demonstrates that it is possible to have a dynamical system which has these features.

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